# CROP INSURANCE IN INCOMPLETE MARKETS

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**Abstract**

*When there are multiple risks threatening the loss of an asset, insurance schemes contingent on one risk alone are incomplete. Such is the case with crop insurance schemes when price variability is uninsured. This paper considers the effect of price risk on crop insurance decisions when price risk is due to supply-and-demand shocks. If demand shocks satisfy the principle of increasing uncertainty, increasing demand uncertainty reduces optimal crop insurance whenever risk aversion is constant or decreasing. In fact, the insurance is so limited for decreasing risk-averse individuals that they strictly prefer those states of the world in which no indemnity is forthcoming to those in which they receive indemnities. Special cases arise when either output risk or demand uncertainty is the sole cause of price risk. In the first case, the optimal insurance is complete even though price variability affects the demand for crop insurance through the correlation between price and output. In the second case, the principle of increasing uncertainty is trivially satisfied and price risk affects optimal insurance levels even though it is independent of output risk.*

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**Key words:** multiple risks, risk markets, price risk, crop insurance, insurance compensation incomplete markets.

In recent years, researchers have directed their attention to the analysis of insurance schemes in economies that do not possess a full set of risk markets. Since complete market economies exist only as allegorical entities, the study of incompleteness enriches the theory of insurance.

This paper examines crop insurance decisions in economies with incomplete risk markets. While economic activity in agriculture is subject to a variety of risks, crop insurance is a claim contingent only on states of output. If the markets for other agricultural risks are absent, insurance decisions have to be taken against the background of uninsurable risks. In this paper, we take price risk to be the principal example of the "other agricultural risks" for which markets may be absent. The objective is to investigate the effect of price risk on crop insurance decisions.

To motivate the key issues, consider the following example. Suppose that the output of a farmer is either one hundred bushels (of say, corn) with probability 0.6, or sixty bushels with probability 0.4. The insurance compensation is the physical loss of 40 bushels valued at the "price election."[1] The problem is to choose the price election (and hence the amount of insurance) at the beginning of the planting season when price and output are unknown. Now compare the choice of price election in two situations that differ only with respect to the extent of price uncertainty. In the first case, the output price is known with certainty to be $2.00 per bushel. In the second case, the output price could be $3.00 per bushel or $1.00 per bushel, with equal probability. So the expected output price is $2.00 per bushel. In which instance, will the farmer buy more insurance? Or could it be that the choice of the price-election is unaffected by price risk?

These questions cannot be resolved in models of agricultural insurance where only output risk is present (Ahsan, Ali, and Kurian (1982), Nelson and Loehman (1987)). It is, however, well known that if price and output are negatively correlated, the resulting natural hedge will reduce the demand for crop insurance. In the above example, though, price and output are independently distributed random variables. Assuming a constant absolute risk aversion coefficient of 0.003, part A of the appendix calculates the optimal insurance to be $80.00 when price is known to be $2.00 but is only $70.40 in the second case when prices are uncertain. Price risk seems to matter even when it is independent of output risk.

So what exactly is the relationship between price risk and crop insurance decisions? To answer the question the next section presents a model in which price variability is caused by output fluctuations and demand shocks. Since crop insurance is contingent only on output risk, demand shocks constitute a source of incompleteness in the insurance scheme. Section 2 examines the effect of incompleteness on the optimal amount of insurance. Two special cases, considered in section 3, arise when either demand uncertainty or output risk is the sole cause of price risk. In the first case, price risk is independent of output risk and for an important class of utility functions, it is shown that price risk affects crop insurance decisions in the manner of the above numerical example. If, on the other hand, price risk is completely induced by output shocks then the optimal insurance is complete even though price variability affects the demand for crop insurance through the correlation between price and output. This is because incompleteness is the consequence of an independent source of risk additional to output risk. In section 4, the change in optimality conditions (as compared to complete insurance schemes) is examined from the point of view of potential moral hazard. The last section discusses the implications of the results and its applicability to other insurance schemes.

## 1. A Model of Incomplete Insurance

### 1.1. Output and Price Distributions

The insurance decision of a risk-averse farmer is considered in a simple stylized world in which price and output risk are the only sources of uncertainty. Suppose that output has a subjective probability distribution of the following form:

$$q = \begin{cases} Q_1 \text{ with probability } (1-\gamma) \\ Q_2 \text{ with probability } \gamma \end{cases} \tag{1}$$

where $q$ is output. Without loss of generality we can assume $Q_1 > Q_2$.

In competitive markets, price variability can be attributed to shifts in either the demand curve or the supply curve or to a combination of the two.[2] Even if supply-and-demand shocks are independent, the subjective probability distributions of output and price need not be independent for an individual farmer. Crop hazards such as drought, floods, and hailstorms affect many farmers in a particular geographical area and so their outputs are likely to be positively correlated. A price-taking farmer may therefore view the probability distribution of price as conditional on output.

Let $\theta$ be a random demand shock independent of the output realization and suppose that it is a two-point distribution taking the value $\theta_1$ with probability $\lambda$ and the value $\theta_2$ with probability $(1 - \lambda)$. Due to these demand shocks the price conditional on an output realization is itself random. More specifically,

$$P|Q_i = \begin{cases} P(Q_i, \theta_1) \text{ with probability } \lambda \\ P(Q_i, \theta_2) \text{ with probability } (1 - \lambda) \end{cases} \quad \text{for } i = 1, 2 \tag{2}$$

where $P(Q_i, \theta_j)$ is the output price conditioned on the output realization $Q_i$ and demand realization $\theta_j$. Without loss of generality, we can assume $\theta_1$ is the favorable demand shock, i.e., $P(Q_i, \theta_1) > P(Q_i, \theta_2)$ for $i = 1, 2$.

The value of output is a random variable $w$ distributed as

$$w = \begin{cases} W_1 \equiv W(Q_1, \theta_1) = P(Q_1, \theta_1)Q_1 \text{ with probability } (1 - \gamma)\lambda \\ W_2 \equiv W(Q_1, \theta_2) = P(Q_1, \theta_2)Q_1 \text{ with probability } (1 - \gamma)(1 - \lambda) \\ W_3 \equiv W(Q_2, \theta_1) = P(Q_2, \theta_1)Q_2 \text{ with probability } \gamma\lambda \\ W_4 \equiv W(Q_2, \theta_2) = P(Q_2, \theta_2)Q_2 \text{ with probability } \gamma(1 - \lambda) \end{cases} \tag{3}$$

Since $\theta_1$ corresponds to the higher price, crop revenue can be order in one of two ways. Either $W_1 > W_2 > W_3 > W_4$ or $W_1 > W_3 > W_2 > W_4$.

### 1.2. Insurance

As the state of the world is the two-tuple $(q, \theta)$ a complete insurance contract would be contingent on $q$ and $\theta$. Since crop insurance contracts are contigent on output alone, the demand shock $\theta$ is the source of incompleteness. Let $I$ be the indemnity, which is payable only if the event $Q_2$ occurs. We suppose the insurance firm is risk neutral and offers actuarially fair insurance.[3] So the premium is $\gamma I$.

Denoting $r$ as the revenue with insurance, we have

$$r = \begin{cases} R_1 \equiv R(Q_1, \theta_1) = P(Q_1, \theta_1)Q_1 - \gamma I \text{ with probability } (1 - \gamma)\lambda \\ R_2 \equiv R(Q_1, \theta_2) = P(Q_1, \theta_2)Q_1 - \gamma I \text{ with probability } (1 - \gamma)(1 - \lambda) \\ R_3 \equiv R(Q_2, \theta_1) = P(Q_2, \theta_1)Q_2 + (1 - \gamma)I \text{ with probability } \gamma\lambda \\ R_4 \equiv R(Q_2, \theta_2) = P(Q_2, \theta_2)Q_2 + (1 - \gamma)I \text{ with probability } \\ \qquad \gamma(1 - \lambda), \end{cases} \tag{4}$$

since $\theta_1$ is associated with a higher price compared to $\theta_2$, $R_1 > R_2$ and $R_3 > R_4$. A complete ordering of incomes is not possible without further information about $I$.

### 1.3. Conditions for Interior Solution

From the set of actuarially fair contracts the optimal insurance is found by maximizing the expectation of an increasing and strictly concave von Neumann-Morgenstern utility function.

$$\begin{aligned}\operatorname*{Max}_{I}\ \eta(I) = {} & (1-\gamma)\lambda U(R_1) + (1-\gamma)(1-\lambda)U(R_2) \\ & + \gamma\lambda U(R_3) + \gamma(1-\lambda)U(R_4)\end{aligned}$$

The optimal level of insurance is positive if $\eta'(I)$ evaluated at $I = 0$ is also positive.[4] Now

$$\begin{aligned}\eta'(I) = {} & \gamma(1-\gamma)\{\lambda U'(R_3) + (1-\lambda)U'(R_4) \\ & - \lambda U'(R_1) - (1-\lambda)U'(R_2)\}\end{aligned} \tag{5}$$

and since $R_k|_{I=0} = W_k$

$$\begin{aligned}\eta'(I)|_{I=0} = {} & \gamma(1-\gamma)\{\gamma(U'(W_3) - U'(W_1)) \\ & + (1-\lambda)(U'(W_4) - U'(W_2))\}\end{aligned} \tag{6}$$

A sufficient condition for $I$ to be positive is for $W_1$ to be greater than $W_3$ and for $W_2$ to be greater than $W_4$. This can be described more compactly by the following notation.

Let $\delta w(\theta) \equiv W(Q_1, \theta) - W(Q_2, \theta)$. $\delta w$ is the revenue loss (for a given state of demand $\theta$) when the output $Q_2$ is realized. Then optimal insurance is positive if $\delta w(\theta_1) > 0$ and $\delta w(\theta_2) > 0$

The above condition simply says that, in the absence of insurance, the farmer's revenue is higher when output is higher irrespective of what demand state is realized. If the price-output correlations are sufficiently negative the condition may not be satisfied and the optimal insurance may be zero or even negative.[5]

### 1.4. The Principle of Increasing Uncertainty

An essential assumption of the analysis is that the random demand shocks are such that $\delta w(\theta_1) > \delta w(\theta_2)$, i.e., the revenue loss is higher in the favorable demand state $\theta_1$.[6] The assumption is equivalent to the *principle of increasing uncertainty*, which may be stated as follows: if increases in output lead to higher expected revenue, then increases in output also lead to greater riskiness of revenue. Thus, if $W(Q_1, \theta)$ has a higher mean revenue than $W(Q_2, \theta)$, then $W(Q_1, \theta)$ is also riskier than $W(Q_2, \theta)$. The equivalence of the assumption $\delta w(\theta_1) > \delta w(\theta_2)$ with the principle of increasing uncertainty is demonstrated in part B of the appendix. The principle of increasing uncertainty was first stated by Leland (1972) who used it to demonstrate the impact of demand uncertainty on firms' output decisions.[7]

## 2. The Effect of Demand Uncertainty

**Proposition 1:** Let $I^*$ be the optimal amount of insurance. If the demand shocks satisfy the principle of increasing uncertainty, $R_1(I^*) > R_3(I^*) > R_4(I^*) > R_2(I^*)$ or equivalently $\delta w(\theta_1) > I^* > \delta w(\theta_2)$.

**Proof:** From (4) note that $R_3 > R_4$ for all **I**. So what needs to be shown is $R_1(I^*) > R_3(I^*)$ and $R_4(I^*) > R_2(I^*)$.

The optimal insurance satisfies

$$\eta'(I^*) = \gamma(1-\gamma)\{\lambda U'(R_3^*) + (1-\lambda)U'(R_4^*) - \lambda U'(R_1^*) - (1-\lambda)U'(R_2^*)\} = 0 \tag{7}$$

where $R_j^*$ denotes $R_j(I^*)$ for $j = 1, \cdots 4$.

Let $\eta_1(I) \equiv U'(R_3) - U'(R_1)$ and $\eta_2(I) \equiv U'(R_2) - U'(R_4)$.

Substituting and rearranging terms, (7) becomes

$$\lambda\eta_1(I^*) - (1-\lambda)\eta_2(I^*) = 0 \tag{8}$$

Clearly $\eta_1(I^*)$ and $\eta_2(I^*)$ must both be of the same sign. Suppose they are both negative.

$$\eta_1 < 0 \Rightarrow R_3^* > R_1^* \Rightarrow W(Q_2, \theta_1) + (1-\gamma)I^* > W(Q_1, \theta_1) - \gamma I^*, \text{ and so } I^* > \delta W(\theta_1). \tag{9}$$

$$\eta_2 < 0 \Rightarrow R_2^* > R_4^* \Rightarrow W(Q_1, \theta_2) - \gamma I^* > W(Q_2, \theta_2) + (1 - \gamma I^*), \text{ and so } I^* < \delta W(\theta_2). \tag{10}$$

Combining the two inequalities, $\delta w(\theta_2) > I^* > \delta w(\theta_1)$. But this contradicts the assumption that $\theta_1$ and $\theta_2$ are such that $\delta w(\theta_1) > \delta w(\theta_2)$. Similarly, $\eta_1$ and $\eta_2$ cannot both be zero. Hence $\eta_1$ and $\eta_2$ are positive. This means $R_3^* < R_1^*$ and $R_2^* < R_4^*$. Since $R_3^* < R_4^*$, we obtain the ordering $R_1^* > R_3^* > R_4^* > R_2^*$. Notice also that the inequalities in (9) and (10) are reversed, and so we obtain upper and lower bounds on the amount of optimal insurance, i.e., $\delta w(\theta_1) > I^* > \delta w(\theta_2)$.

The complete ordering of the $R_j$'s is a direct consequence of the first-order condition and the principle of increasing uncertainty. Recall, that in the absence of insurance, we know that either $W_1 > W_2 > W_3 >$

$W_4$ or $W_1 > W_3 > W_2 > W_4$. In either case the worst income state is $W_4$ when both price and output are low. With insurance, however, the ordering changes in a significant way. Now the worst income state is $R_2$, when price is low but output is high. In this state, premium payments have to be made, even though the farmer suffers losses due to low prices. For the farmer, the incomplete nature of insurance creates a difficult trade-off between output and price risks. While output risks are clearly reduced, the farmer is worse off in the low price-high output state $R_2$. Further the fact that $R_2$ *decreases with greater pruchase of insurance, suggests that* $I^*$ cannot be too high. The argument is made more precise in the following propositions.

**Proposition 2:** If the principle of increasing uncertainty is satisfied and if risk aversion is nonincreasing, the optimal insurance is strictly less than expected value of crop loss.

**Proof:** The proof consists in showing $\eta'(I)$ to be negative for all values of I greater than or equal to the expected value of crop loss.[8]

Let $\bar{p}|Q_1$ denote the expected price when output is $Q_1$ and let $\bar{p}|Q_2$ denote the expected price when output is $Q_2$. Then the expected value of crop loss is $E[\delta w(\theta)] = (\bar{p}|Q_1)Q_1 - (\bar{p}|Q_2)Q_2$.

Let $r_G(\theta)$ denote the random income in the high crop output "good" states and $r_B(\theta)$ the random income in the low crop output "bad" states, i.e.,

$$r_G(\theta) = (p|Q_1)\,Q_1 - \gamma I$$
$$= \begin{cases} R_1 = P(Q_1, \theta_1)\,Q_1 - \gamma I \text{ with probability } \lambda \\ R_2 = P(Q_1, \theta_2)\,Q_1 - \gamma I \text{ with probability } (1-\lambda) \end{cases} \tag{11}$$

and

$$r_B(\theta) = (p|Q_2)\,Q_2 + (1 - \gamma I)$$
$$= \begin{cases} R_3 = P(Q_2, \theta_1)\,Q_2 + (1-\gamma)\,I \text{ with probability } \lambda \\ R_4 = P(Q_2, \theta_2)\,Q_2 + (1-\gamma)\,I \text{ with probability } (1-\lambda) \end{cases} \tag{12}$$

Then from (5), (11) and (12), $\eta'(I)$ can be written more compactly as

$$\eta'(I) = \gamma(1-\gamma)\{E^\theta U'(r_G(\theta)) - E^\theta U'(r_B(\theta))\} \tag{13}$$

where the superscript on the expectations operator denotes that the expectations are with respect to the distribution of demand shock $\theta$. The sign of $\eta'(I)$ therefore, depends on the difference in expected marginal utilities between the low and high crop states. Now, $r_G(Q_1, \theta) -$

$r_B(Q_2, \theta) = \delta w(\theta) - I$. So $E^{\theta}U'(r_G(\theta)) = E^{\theta}U'(r_B(\theta) + \delta w(\theta) - I)$. Let $v(\theta) = \delta w(\theta) - E[\delta w(\theta)]$. Clearly $E^{\theta}v(\theta) = 0$ and since $\delta W(\theta_1) > \delta W(\theta_2)$, $v(\theta_1) > 0$ and $v(\theta_2) < 0$.

Substituting, $EU'(r_G) = EU'(r_B + v + E[\delta w(\theta)] - I)$.

If $I \geqslant E[\delta w(\theta)]$, $E^{\theta}U'(r_G) \geqslant E^{\theta}U'(r_B + v) > E^{\theta}U'(r_{BL})$ where the second inequality follows from the convexity of marginal utility ($U''' > 0$)[9] and from the observation that $r_B + v$ is a mean preserving spread of $r_G$. Therefore, the optimal insurance is less than the expected value of crop loss.

The result for the certainty case is the following:

**Proposition 3:** If the price conditional on an output realization is certain and equal to its conditional expectation, then the optimal insurance is equal to the expected value of crop loss.

**Proof:** If demand uncertainty vanishes such that $P(Q_1, \theta_1) = P(Q_1, \theta_2) = \bar{p}|Q_1$ and $P(Q_2, \theta_1) = P(Q_2, \theta_2) = \bar{p}|Q_2$, then $R_1 = R_2$ and $R_3 = R_4$. First-order condition (7) reduces to $U'(R_1) = U'(R_3)$. Therefore, $I^* = (\bar{p}|Q_1)Q_1 - (\bar{p}|Q_2)Q_2$.

If we refer to $(\bar{p}|Q_1)Q_1 - (\bar{p}|Q_2)Q_2$ as the certainty level of insurance, then proposition 2 proves that optimal level of insurance under demand uncertainty is less than the certainty level of insurance. The next proposition is concerned with the marginal impact of demand uncertainty, i.e., the effect of making the conditional price distribution slightly more risky.

**Proposition 4:** If the principle of increasing uncertainty is satisfied and if risk aversion is constant or decreasing, an increase in demand uncertainty reduces optimal insurance.

The proof is in part C of the appendix.

## 3. Two Special Cases

In the earlier section, price variability was the consequence of supply and demand shocks, i.e., $p = p(q, \theta)$. Two special cases of this formulation are when $p = p(q)$ and $p = p(\theta)$. In the first case, all the price variability is due to output fluctuations. In the second case, all price variability is due to demand uncertainty, which leads to price risk being independent of output risk.

### 3.1. Price Risk Due to Output Variability

Output $q$ has a two outcome distribution given by (1). If there is no demand risk, price is also a two-outcome distribution with

$$p = \begin{cases} P(Q_1) \text{ with probability } (1-\gamma) \\ P(Q_2) \text{ with probability } \gamma \end{cases} \tag{14}$$

Then it is not difficult to see that $I^* = P(Q_1)Q_1 - P(Q_2)Q_2$, i.e., the optimal amount of insurance is simply the value of the crop loss.[10]

Clearly, price variability affects the optimal value of insurance. More specifically, the optimal insurance depends on the correlation between price and output. But the optimal insurance is not incomplete. There is only one source of risk in the model. Price variability is completely induced by supply variability for which insurance is available. So the insurance scheme is complete. The incompleteness in crop insurance is not a consequence of price variability per se but rather it is due to the presence of a source of uninsurable risk (demand uncertainty) different from output risk.

### 3.2. Price Risk Due to Demand Uncertainty

Let $P(\theta_1) \equiv P_1$ and $P(\theta_2) \equiv P_2$. Then

$$p = \begin{cases} P_2 \text{ with probability } \lambda \\ P_2 \text{ with probability } (1-\lambda) \end{cases} \tag{15}$$

where without loss of generality, we assume $P_1 > P_2$.

If price and output are independent, it is not necessary to assume the principle of increasing uncertainty as it is trivially true.[11] So by proposition 2, the optimal insurance is less than the expected revenue loss for all constant and decreasing risk aversion utility functions. This explains the comparisions of optimal insurance levels in the example in the introduction.

## 4. Incentive Implications

The effect of demand uncertainty on the optimal crop insurance can also be seen in another way. In the absence of demand shocks, it is optimal to equalize the farmer's income across the two output states of nature. The individual is consequently indifferent between them. What can we infer when insurance is incomplete, i.e., is $E^{\theta}U(r_G) \gtreqless E^{\theta}U(r_B)$?

**Proposition 5:** If the demand shocks satisfy the principle of increasing uncertainty, then

(i) $E^{\theta}U(r_G(\theta)) > E^{\theta}U(r_B(\theta))$ for decreasing absolute risk aversion utility functions

(ii) $E^{\theta}U(r_G(\theta)) = E^{\theta}U(r_B(\theta))$ for constant absolute risk aversion utility functions

(iii) $E^{\theta}U(r_G(\theta)) < E^{\theta}U(r_B(\theta))$ for increasing absolute risk aversion utility functions

**Proof:** Define the inverse of the marginal utility function by $z : z \equiv (U')^{-1}$. Also define $\phi$ as the composite function of $U$ and $z : \phi = U \circ z$. Denoting by $\alpha$ a value of marginal utility of income the following relation holds $\phi(\alpha) = U(z(\alpha))$.

**Lemma:** $\phi''(\alpha)$ is greater than, equal to, or less than zero depending on whether the utility function exhibits decreasing, constant, or increasing absolute risk aversion.

**Proof of lemma:** See Imai, Geanakoplos, and Ito (1981).

From the first-order condition $E^{\theta}U'(r_G) = E^{\theta}U'(r_B)$. From proposition 1, we also know $U'(R_2^*) > U'(R_4^*) > U'(R_3^*) > U'(R_1^*)$. At the optimum, therefore, $U'(r_G)$ is a mean preserving spread of $U'(r_B)$. But if absolute risk aversion is decreasing, $\phi$ is a strictly convex function of marginal utility (from lemma). Therefore, by Jensen's inequality, $E^{\theta}\phi(U'(r_G)) = E^{\theta}U(r_G) > E^{\theta}U(r_B) = E^{\theta}\phi(U'(r_B))$. The proof is similar for other cases.

When individuals are decreasing risk-averse, incompleteness limits the amount of insurance so much that the individual prefers ex ante the high output state. If we allow for moral hazard so that the individual's actions affect the probabilities of high and low output, then the incentive problem is least serious for decreasing risk-averse individuals simply because their insurance coverage is already curtailed by price risk.[12]

## 5. Concluding Remarks

This paper has explored some properties of incomplete crop insurance schemes. It is well known that actuarially fair complete insurance results in equalization of marginal utilities and hence net incomes across different states of nature. Incomplete insurance schemes, by contrast, cannot stabilize incomes completely. This is reflected in the first-order condition that requires the expected marginal utilities to be equal across the high and low output states. Since the marginal costs of insurance are evaluated at different utility levels corresponding to the different price states, the demand for insurance is, in many circumstances, sensitive to price risk. The farmer buying crop insurance risks greater exposure to the uninsured price variability, so the demand for crop insurance emerges from a tradeoff between output and price risks.

The implication is that introducing the market for price risks would increase the demand for crop insurance. In a study of crop insurance in the United States, Gardner and Kramer (1986) state that a "general reason for coordinating price support programs and output insurance programs is that either one by itself may have only small risk reducing effects in the absence of the other. This situation arises when price fluctuations are caused by random output variations that are correlated across farms since in this situation, low yields tend to be compensated by higher prices." As we have seen, the above argument could be made even if price and output shocks are independently distributed. Further, the analysis can be extended to show that introducing futures markets increases the optimum level of crop insurance (see Ramaswami and Roe, (1989)).

In the analysis it was convenient to assume actuarially fair insurance. If the premium includes a loading fee or a subsidy the results need to be qualified. The key result is proposition 1, which states that if insurance is actuarially fair, the optimal insurance is bounded below by the revenue loss in the low-price state and bounded above by the revenue loss in the high-price state. However, if the premium is higher than actuarial cost, the optimal insurance may be less than the lower bound and if the premium is below the actuarial cost, the optimal insurance may be greater than the upper bound. So whether proposition 1 is violated depends on the extent of mark-up or subsidy on the actuarial cost.[13]

The results of this paper apply to all situations where the insurable and the uninsurable risks interact multiplicatively.[14] Earlier Doherty and Schlesinger (1983) have studied the properties of optimal insurance when the multiple risks are additive. An implication of their analysis is that uninsurable risks are inconsequential for the demand for insurance if the multiple risks are independent. This, as we have seen, is not true in the multiplicative case. The multiplicative specification of risks arises quite naturally in contexts where the assets that are insured against physical loss are also subject to uninsured fluctuations in unit value. Of course, real-world insurance schemes may also occur in settings that satisfy neither the additive nor the multiplicative specification.[15] For this reason, it would be worthwhile in future investigations to consider more general structures capable of accomodating a variety of cases.

## Appendix

### Part A

From (7) and (12) the optimal insurance satisfies $E^{\theta}U'(r_G) = E^{\theta}U'(r_B)$. In the example price risk is independent of output risk. For the constant risk aversion case the first-order condition reduces to $Ee^{-Ar_G} = Ee^{-Ar_B}$ where the expectations are with respect to price and $A$ is the risk aversion coefficient. Substituting for $r_B = pQ_2 + (1 - \gamma)I$ and $r_G = pQ_1 - \gamma I$, we obtain

$$e^{A\gamma I}\, E(e^{-ApQ_1}) = e^{-A(1-\gamma)I}\, E(e^{-ApQ_2})$$

Solving for $I$,

$$I^* = -1/A[\ln(Ee^{-ApQ_1}/Ee^{-ApQ_2})]$$

Using a value of $A = .003$, $I^* = \$80.00$ when $P_1 = P_2 = \$2.00$, and $I^* = \$70.40$ when $P_1 = \$3.00$ and $P_2 = \$1.00$ with equal probability.

### Part B

We wish to show that the assumption $\delta w(\theta_1) > \delta w(\theta_2)$ is equivalent to the principle of increasing uncertainty. According to the principle, if $W(Q_1, \theta)$ has a higher mean revenue than $W(Q_2, \theta)$, then $W(Q_1, \theta)$ is also riskier than $W(Q_2, \theta)$.

Suppose $EW(Q_1, \theta) > EW(Q_2, \theta)$. Then we need to show that if $\delta w(\theta_1) > \delta w(\theta_2)$ then $W(Q_1, \theta)$ is riskier than $W(Q_2, \theta)$ after adjusting for the difference in means, i.e.,

(a) $W(Q_1, \theta_1) > W(Q_2, \theta_1) + EW(Q_1, \theta) - EW(Q_2, \theta)$ and

(b) $W(Q_1, \theta_2) < W(Q_2, \theta_2) + EW(Q_1, \theta) - EW(Q_2, \theta)$.

Consider the right-hand side of (a):

$$\begin{aligned} &W(Q_2, \theta_1) + EW(Q_1, \theta) - EW(Q_2, \theta)\\ &\quad = W(Q_2, \theta_1) + \lambda(W(Q_1, \theta_1) - W(Q_2, \theta_1)) + (1 - \lambda)(W(Q_1, \theta_2)\\ &\qquad - W(Q_2, \theta_2))\\ &\quad = (1 - \lambda)(\delta w(\theta_2) - \delta w(\theta_1)) + W(Q_1, \theta_1)\\ &\quad < W(Q_1, \theta_1). \end{aligned}$$

Consider the right-hand side of (b):

$$\begin{aligned} &W(Q_2, \theta_2) + EW(Q_1, \theta) - EW(Q_2, \theta) \\ &\quad = W(Q_2, \theta_2) + \lambda(W(Q_1, \theta_1) - W(Q_2, \theta_1)) + (1 - \lambda)(W(Q_1, \theta_2) \\ &\qquad - W(Q_2, \theta_2)) \\ &\quad = \lambda(\delta w(\theta_1) - \delta w(\theta_2)) + W(Q_1, \theta_2) \\ &\quad > W(Q_1, \theta_2). \end{aligned}$$

Since each of the above steps can be reversed, the reverse implication is also true.

## Part C

**Proof of Proposition 4.** The initial conditional price distribution can be represented as

$$p\,|\,Q_i = \begin{cases} P(Q_i, \theta_1) - k_1 \text{ with probability } \lambda \\ P(Q_i, \theta_2) + k_2 \text{ with probability } (1 - \lambda) \end{cases} \qquad i = 1, 2$$

where $k_1 = k_2 = 0$ (initially). Now an increase (decrease) in the riskiness (in the Rothschild-Stiglitz 1970 sense) of the $p\,|\,Q_i$ distribution is represented by a decrease (increase) in $k_2$, $\bar{p}\,|\,Q_i$ held constant. Since $\bar{p}\,|\,Q_i = \lambda[P(Q_i, \theta_1) - k_1] + (1 - \lambda)[P(Q_i, \theta_2) + k_2]$, $dk_1/dk_2 = (1 - \lambda)/\lambda$. As $\eta''(I) < 0$, $\partial I^*/\partial k_2$ is of the same sign as $\partial\eta'(I)/\partial k_2$ evaluated at $I^*$.

$$\begin{aligned} \partial\eta'(I)/\partial k_2|_{I^*} = \gamma(1 - \gamma)\{&\lambda(U''(R_3^*)Q_2 - U''(R_1^*)Q_1)dk_1/dk_2 \\ &- (1 - \lambda)(U''(R_2^*)Q_1 - U''(R_4^*)Q_2)\} \end{aligned}$$

Substituting for $dk_1/dk_2$

$$\begin{aligned} \partial\eta'(I)/\partial p_2|_{I=I^*} &= \gamma(1 - \gamma)(1 - \lambda)\{U''(R_1^*)Q_1 - U''(R_3^*)(Q_1 - (Q_1 - Q_2)) \\ &\qquad + U''(R_4^*)(Q_1 - (Q_1 - Q_2)) - U''(R_2^*)Q_1\}. \\ &= \gamma(1 - \gamma)(1 - \lambda)\{(U''(R_1^*) - U''(R_3^*))Q_1\} \\ &\qquad + (U''(R_4^*) - U''(R_2^*))Q_1 + (U''(R_3^*) \\ &\qquad - U''(R_4^*))(Q_1 - Q_2)\}, \end{aligned}$$

which is strictly positive because $R_1^* > R_3^* > R_4^* > R_2^*$ and $U''' > 0$.

## Notes

1. The price-election is the price at which the insurance company compensates the farmer for a unit loss of the commodity. In this United States the current crop insurance

practice is to offer a farmer buying insurance a choice between three price elections. For a description of the crop insurance program, see the report by the U.S. General Accounting Office (1984).

2. See Newbery and Stiglitz (1981), ch. 4, for a discussion about the causes of price variability in agricultural commodity markets.

3. Considering that the government is often the principal insurer in agriculture, this is not an unreasonable assumption. The consequences of departing from this assumption are indicated later. In the United States the administrative expenses of the Federal Crop Insurance Corporation are excluded from premium calculations and premiums are priced below actuarial cost. The subsidy depends on the level of deductible but is not greater than 30% of the actuarial cost.

4. As noted later, strict concavity of the utility function guarantees the strict convacity of $\eta$ in $I$.

5. It is not essential for the analysis that the optimal insurance be positive.

6. Another way of saying this is that $\theta$ induces the same ordering over revenue losses as over prices.

7. Sandmo (1971) analysed the impact of price uncertainty on a competitive firm's output decisións. Leland extended the analysis to consider the impact of demand uncertainty on a monopolist firm. As generalized by Coes (1977), the principal result is that if stochastic demand shocks satisfy the *principle of increasing uncertainty*, increases in demand uncertainty reduce the optimal output of a decreasing risk-averse firm.

8. This is enough since $\eta$ is strictly concave in $I$. From (8), $\eta''(I) = \lambda\eta_1'(I) - (1 - \lambda)\eta_2'(I)$ where $\eta_1'(I) = (1 - \gamma)U''(R_3) + \gamma U''(R_1) < 0$ and $\eta_2'(I) = -(1 - \gamma)U''(R_4) - \gamma U''(R_2) < 0$.

9. All constant and decreasing absolute risk aversion utility functions have a strictly positive third derivative. It can also be shown that the optimum insurance is equal to the expected value of crop loss when marginal utility is linear and greater than the expected value of crop loss when marginal utility is concave. Therefore, convexity of marginal utility is a necessary and sufficient condition for demand uncertainty to reduce optimal insurance.

10. This is the result of proposition 3 in which the prices are assumed to be certain at their expected values.

11. $\delta W(\theta_1) = P_1(Q_1 - Q_2) > P_2(Q_1 - Q_2) = \delta W(\theta_2)$.

12. Imai, Geanakoplos, and Ito report a result opposite to ours. They consider unemployment insurance schemes where severance payments are made to laid-off workers. But "since severance payments usually do not depend on outcomes at alternative opportunities after layoff they are considered at best incomplete insurance for layoff" (Ito (1986)). The issue that is investigated is whether the laid-off worker could be better off, in an ex-ante sense, than the retained worker. This is indeed the case for individuals with decreasing risk-averse utility functions. Individuals with constant risk-aversion utility functions are indifferent between the two states while increasing risk-averse individuals prefer to be retained. So, the incentive problem is most serious for decreasing risk-averse individuals. The difference between their model and ours lies in the asumption about the uninsured variable. The uncertainty in the rehiring wage, in the Imai model, affects only the marginal benefit of insurance (the expected marginal utility if the worker is laid off) and not the marginal cost (the marginal utility of the *sure* wage net of premium payments). See Ito and Machina (1983), and Ito for further variants of the problem.

13. Suppose C is the actuarial cost of insurance. Then, by continuity arguments, there exists insurance prices $C_1 > C > C_2$ such that proposition 1 is true for all premiums that lie in the interval $[C_1, C_2]$.

14. Some examples of incomplete insurance for multiplicative risks are
(a) when a firm faces price and output uncertainty but can obtain insurance only against price risks,
(b) when an exporter can obtain insurance against exchange rate risks but not against fluctuations in world market price, and
(c) when a work of art can be insured against loss of theft but not against changes in its market value. This example is due to Turnbull (1983).

15. The unemployment insurance scheme considered by Imai, Geanakoplos, and Ito is an example.

## References

Ahsan, S. M., A. A. G. Ali, and N. Kurian. (1982). "Towards a Theory of Agricultural Insurance," *American Journal of Agricultural Economics* 64, 520–529.

Coes, D. V. (1977). "Firm Output and Changes in Uncertainty," *American Economic Review* 67, 249–251.

Doherty, N., and H. Schlesinger. (1983). "Optimal Insurance in Incomplete Markets," *Journal of Political Economy* 91, 1045–1054.

Gardner, B., and R. Kramer. (1986). "Experience with Crop Insurance Programs in the United States" in *Crop Insurance for Agricultural Development*, P. Hazell, C. Pomareda and A. Valdes, eds. Baltimore: the Johns Hopkins University Press.

Imai, H., J. Geanakoplos, and T. Ito. (1981). "Incomplete Insurance and Absolute Risk Aversion," *Economics Letters* 8, 107–112.

Ito, T. (1986). "Implicit Contracts and Risk Aversion," in *Equilibrium Analysis, Essays in Honor of Kenneth J. Arrow*, Volume II, W. P. Heller, R. M. Starr and D. P. Starrett, eds. Cambridge University Press.

Ito, T., and M. Machina. (1983). "The Incentive Implications of Incomplete Insurance: The Multiplicative Case," *Economics Letters* 13, 319–323.

Leland, H. E. (1972). "Theory of the Firm Facing Uncertain Demand," *American Economic Review* 62, 278–291.

Nelson, C., and E. T. Loehman. (1987). "Further Toward a Theory of Agricultural Insurance," *American Journal of Agricultural Economics* 69, 523–531.

Newberry, D. M. G., and J. E. Stiglitz. (1981). *The Theory of Commodity Price Stabilization*, New York: Oxford University Press.

Ramaswami, B. and T. Roe. (1989). "Incompleteness in Insurance: An Analysis of the Multiplicative Case," Bulletin No. 89–7, Economic Development Center, University of Minnesota.

Rothschild, M., and J. Stiglitz. (1970). "Increasing Risk I: A Definition," *Journal of Economic Theory* 2, 225–243.

Sandmo, A. (1971). "On the Theory of the Competitive Firm under Price Uncertainty," *American Economic Review* 61, 65–73.

Turnbull, S. M. (1983). "Additional Aspects of Rational Insurance Purchasing," *Journal of Business* 56, 217–229.

U.S. General Accounting Office. (1984). "More Attention Needed in Key Areas of the Expanded Crop Insurance Program," Report to the Secretary of Agriculture, GAO/RCED-84-65, March 14, Washington, D.C.